HW 5 Help

34. **ORGANIZE AND PLAN** We'll calculate initial and final momentum using $p = mv$ to get the change in momentum. Once we know Δp we'll use the impulse-momentum theorem $J = F\Delta t = \Delta p$ to find force.
Known: $m = 1120\ \text{kg};\ \ v_i = 5.0\ \text{m/s};\ \ v_f = 15\ \text{m/s};\ \ \Delta t = 12\ \text{s}.$

SOLVE (a) The initial and final momenta are:

$$p_i = mv_i$$

$$p_f = mv_f$$

$$\Delta p = p_f - p_i = m(v_f - v_i) = 1120\ \text{kg}(15\ \text{m/s} - 5.0\ \text{m/s}) = 11200\ \text{kg}\cdot\text{m/s}$$

(b) To find the required force, we use

$$\Delta p = J = F\Delta t$$

$$F = \frac{\Delta p}{\Delta t} = \frac{11200\ \text{kg}\cdot\text{m/s}}{12\ \text{s}} = 930\ \text{N}$$

REFLECT The units of momentum, $\text{kg}\cdot\text{m/s}$ and force, N, come out correctly in (a) and (b). The force of $930\ \text{N}$ is provided by the reaction force of the road surface on the tires of the car.

38. **ORGANIZE AND PLAN** We find the magnitude of the impulse from the change in momentum for the car from moving to being at rest, using $p = mv.$ Once we know the change in momentum, we can use $\Delta p = F\Delta t$ to find the average force of the wall on the car.
Known: $v_i = 17.9\ \text{m/s};\ \ v_f = 0\ \text{m/s};\ \ \Delta t = 0.275\ \text{s};\ \ m = 1240\ \text{kg}.$

SOLVE (a) First, to find impulse,

$$J = \Delta p = m(v_f - v_i) = (1240\ \text{kg})(-17.9\ \text{m/s})$$

$$J = 22200\ \text{kg}\cdot\text{m/s}$$

(b) Then, using $\Delta p = F\Delta t,$

$$F = \frac{\Delta p}{\Delta t} = \frac{22200\ \text{kg}\cdot\text{m/s}}{0.275\ \text{s}} = 80700\ \text{N}$$

REFLECT The wall is not exactly stationary! Newton's third law tells us that it exerts a force on the car as long as the car exerts a force on the wall during the process of colliding. The wall just does not move very far compared to the car.

44. **ORGANIZE AND PLAN** This starts out as a one-dimensional problem and ends up as a two-dimensional problem, so we'll use vector-space notation in two dimensions. We calculate the initial momenta in each direction and add the impulse in the *y*-direction to get final momentum. We'll also express the answer as a resultant vector.

Known: $m = 150.\text{g}$ $\vec{v}_i = 0.45 \text{ m/s}\hat{i}$ $\Delta t = 1.5 \text{ s}$; $\vec{F} = -0.15 \text{ N}\hat{j}$.

SOLVE First, convert mass to kilograms.

$$m = 150.\text{g}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = 0.150 \text{ kg}$$

(a) Now we find the change in momentum,

$$\Delta\vec{p} = \vec{J} = \vec{F}\Delta t = \vec{F}_x\Delta t + \vec{F}_y\Delta t = 0\hat{i} + (-0.15 \text{ N})(1.5 \text{ s})\hat{j} = (-0.225 \text{ kg}\cdot\text{m/s})\hat{j}$$

(b) Now we find the final momentum using the result from (a),

$$\vec{p}_f = \vec{p}_i + \vec{J} = m\vec{v}_i + \vec{J} = \left(m\vec{v}_x + \vec{J}_x\right) + \left(m\vec{v}_y + \vec{J}_y\right)$$

$$\vec{p}_f = (0.150 \text{ kg})(0.45 \text{ m/s})\hat{i} + (0 \text{ N}\cdot\text{s})\hat{i} + (0 \text{ kg}\cdot\text{m/s})\hat{j} + (-0.225 \text{ kg}\cdot\text{m/s})\hat{j}$$

$$\vec{v}_f = \frac{\vec{p}}{m} = (0.45 \text{ m/s})\hat{i} + (1.50 \text{ m/s})\hat{j}$$

Putting this in resultant-vector format,

$$v_f = \sqrt{(0.45 \text{ m/s})^2 + (1.50 \text{ m/s})^2} = 1.57 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{1.50}{0.45}\right) = 73.3°$$

$\vec{v}_f = 1.57 \text{m/s}$ at 73.3° above the positive *x*-axis.

REFLECT The ball begins by moving with constant velocity in the *x*-direction. As the constant force is applied in the *y*-direction, the ball accelerates in that direction, following a parabolic curve. At the end of the 1.5 seconds, the ball continues to the right and upward at an angle of 73.3° above the positive *x*-axis.

60. **ORGANIZE AND PLAN** Since the two spacecraft are originally separate and are joined after the collision, this is a perfectly inelastic collision. We'll convert tonnes to kilograms. Then we'll use $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$ to find the final velocity of the joined spacecraft.

Known: $m_1 = 95$ tonne; $m_2 = 75$ tonne; $v_{1i} = 0.34$ m/s; $v_2 = -0.58$ m/s.

SOLVE First convert tonne to kilogram:

$$m_1 = 95 \text{ tonne}\left(\frac{1000 \text{ kg}}{1 \text{ tonne}}\right) = 9.5\times10^4 \text{ kg}$$

Likewise,

$$m_2 = 7.5\times10^4 \text{ kg}$$

For conservation of momentum,

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(9.5\times10^4 \text{ kg})(0.34 \text{ m/s}) + (7.5\times10^4 \text{ kg})(-0.58 \text{ m/s})}{9.5\times10^4 \text{ kg} + 7.5\times10^4 \text{ kg}}$$

$$v_f = -0.066 \text{ m/s}$$

REFLECT The momenta of the two craft are nearly evenly matched. We can't predict without doing the math which direction the joined craft will finally be traveling. It turns out that their final velocity will be about 2.6 in/s in the original direction of the less massive spacecraft.

68.**ORGANIZE AND PLAN** We'll use conservation of mechanical energy to convert the gravitational potential energy U_g of the bullet and block to kinetic energy using $mgh = \frac{1}{2}mv^2$. From kinetic energy, we can find the velocity of the block just after the bullet embeds itself in it. From the velocity of this combined block/bullet system and the fact that the block was originally stationary, we will find the original velocity of the bullet through conservation of momentum.

Known: $m_{\text{bullet}} = 9.72\text{g} = 9.72 \times 10^{-3}\ \text{kg};\quad m_{\text{block}} = 4.60\ \text{kg};\quad h = 16.8\ \text{cm} = 0.168\ \text{m}.$

SOLVE Rearrange to find initial velocity of the block/bullet system:

$$mgh = \tfrac{1}{2}m_{xf}v^2$$

$$v_{xf} = \sqrt{2gh} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(0.168\ \text{m}\right)} = 1.81\ \text{m/s}$$

Then knowing that the collision was perfectly inelastic,

$$m_{\text{bullet}}v_{\text{bullet,xi}} = \left(m_{\text{bullet}} + m_{\text{block}}\right)v_{xf}$$

$$v_{\text{bullet,xi}} = \left(\frac{m_{\text{bullet}} + m_{\text{block}}}{m_{\text{bullet}}}\right)v_{xf} = \left(\frac{9.72 \times 10^{-3}\ \text{kg} + 4.60\ \text{kg}}{9.72 \times 10^{-3}\ \text{kg}}\right)1.81\ \text{m/s} = 861\ \text{m/s}$$

REFLECT The greater the velocity of the bullet, the higher the pendulum rises. All the momentum of the bullet is conserved in the bullet/block system. Our answer is about $2800\ \text{ft/s}$ which is roughly the speed of a .30-caliber rifle bullet. The mass of $9.72\ \text{g}$ makes it exactly a 150-grain bullet, so we have a reasonable answer.

74.**ORGANIZE AND PLAN** Here we must find initial and final kinetic energy and their ratio. Mass is not given, but it will cancel when we calculate the ratio of kinetic energy. We're asked to find the coefficient of restitution (COR).
Known: $v_i = 26.0 \text{ m/s}; \; v_f = -21.0 \text{ m/s}.$

SOLVE We'll express COR as the ratio of final to initial kinetic energy:

$$\text{COR} = \frac{K_f}{K_i} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \frac{v_f^2}{v_i^2} = \frac{(-21.0 \text{ m/s})^2}{(26.0 \text{ m/s})^2} = 0.652$$

REFLECT We notice that we do not have to calculate momentum in this problem. Moreover, since COR is a pure number, any consistent units for energy or velocity or will give the correct answer.

82. **ORGANIZE AND PLAN** We establish our coordinate system as the frame of reference, with the bomb stationary with respect to the origin. We'll set the final momenta of the pieces equal to zero. Since the pieces are of the same mass, this quantity will cancel in our calculations. Using vector addition, we can find the velocity of the third piece.
Known: $m_1 = m_2 = m_3 = m; \; \vec{v}_1 = (13.4 \text{ m/s})\hat{j}; \; \vec{v}_2 = (-16.1 \text{ m/s})\hat{i}.$

SOLVE Setting the initial and final momenta equal to zero,

$$\vec{p}_f = \vec{p}_i = 0 \text{ kg} \cdot \text{m/s}$$

$$m_1\vec{v}_1 + m_1\vec{v}_2 + m_1\vec{v}_3 = 0 \text{ kg} \cdot \text{m/s}$$

Since the masses are equal,

$$\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = 0 \text{ kg} \cdot \text{m/s}$$

$$\vec{v}_3 = -\vec{v}_1 - \vec{v}_2 = (+16.1 \text{ m/s})\hat{i} - (13.4 \text{ m/s})\hat{j}$$

$$v_3 = \sqrt{(16.1 \text{ m/s})^2 + (-13.4 \text{ m/s})^2} = 20.9 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{-13.4}{16.1}\right) = -39.8°$$

This is in quadrant IV at 39.8° below the positive *x*-axis.
REFLECT Graphically, it makes sense that the momentum of one faster piece at an angle in the fourth quadrant is equal to the sum of the momenta of two slower pieces at right angle on the axes.

88. **ORGANIZE AND PLAN** In Figure P6.88 in the text, we are given three of the four velocities and can calculate the fourth from $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$. Once we know the initial and final velocities we can calculate initial and final kinetic energies. If these are equal, then it was an elastic collision.

Known: $m_1 = m_2$; $v_{1i} = 0.250\ \text{m/s}$ directed 45° above the positive *x*-axis; $v_{2i} = 0.250\ \text{m/s}$ directed 45° above the negative *x*-axis; $v_{1f} = 0.290\ \text{m/s}$ directed 30° to the left of the positive *y*-axis.

SOLVE (a) For conservation of momentum,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Since $m_1 = m_2$,

$$\vec{v}_{1i} + \vec{v}_{2i} = \vec{v}_{1f} + \vec{v}_{2f}$$

$$\vec{v}_{2f} = \vec{v}_{1i} + \vec{v}_{2i} - \vec{v}_{1f}$$

$$\vec{v}_{2f} = (0.250\ \text{m/s})(\sin 45°)\hat{i} - (0.250\ \text{m/s})(\sin 45°)\hat{i} + (0.250\ \text{m/s})(\cos 45°)\hat{j}$$
$$- (0.290\ \text{m/s})(\sin 30°)\hat{j} + (0.290\ \text{m/s})(\cos 30°)\hat{j}$$

$$\vec{v}_{2f} = (0.145\text{m/s})\hat{i} + (0.102\text{m/s})\hat{j}$$

(b) Since each term in the kinetic energy equation contains an equal mass, we can simply compare the squares of the velocities to see of kinetic energy is conserved. To find kinetic energy, we only need the square of v_{2f}.

$$v_{2f}^2 = (0.145\ \text{m/s})^2 + (0.102\ \text{m/s})^2 = 0.0314\ \text{m}^2/\text{s}^2$$

$$v_{1i}^2 + v_{2i}^2 = (0.250\ \text{m/s})^2 + (0.250\ \text{m/s})^2 = 0.125\ \text{m}^2/\text{s}^2$$

But

$$v_{1f}^2 + v_{2f}^2 = (0.290\ \text{m/s})^2 + 0.0314\ \text{m}^2/\text{s}^2 = 0.115\ \text{m}^2/\text{s}^2$$

Therefore $K_i \neq K_f$ and the collision is nearly, but not perfectly elastic.

REFLECT The coefficient of restitution in this collision is $\frac{0.115\ \text{m}^2/\text{s}^2}{0.125\ \text{m}^2/\text{s}^2} = 0.92$ or about that of a bouncing Superball®.

92.**ORGANIZE AND PLAN** Here we have a two-particle system. We'll measure from the center of the carbon atom to the center of the oxygen atom (the bond length) and use the formula $X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$ to find the center of mass. We'll use the subscript C for carbon and O for oxygen.

Known: $m_C = 1.995\times10^{-26}$ kg; $m_O = 2.657\times10^{-26}$ kg; $x_O = 0.112$ nm $= 1.12\times10^{-10}$ m.

SOLVE We use the center of mass formula,

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_C x_C + m_O x}{m_C + m_O} - \frac{0 + (2.657 \text{ x } 10^{-26}\text{ kg})(1.12\times10^{-10}\text{ m})}{1.995\times10^{-26}\text{ kg} + 2.657\times10^{-26}\text{ kg}} = 6.40\times10^{-11}\text{ m}$$

X_{cm} is 6.40×10^{-11} m, or 0.064 nm from the center of the carbon atom.

REFLECT Since an oxygen atom is more massive than a carbon atom, we expect the center of mass to be closer to the oxygen atom, and our answer is reasonable.

94.**ORGANIZE AND PLAN** We'll model the meterstick as a point object with its center of mass at its geometric center. Then we'll use that fact to find the new center of mass with the two additional objects in (b). We'll use the center-of-mass formula $X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$.

Subscript 1 will be for the meterstick, and subscripts 2 and 3 for the masses.

Known: $m_1 = 120.\text{g} = 0.120$ kg; $m_2 = 100.\text{g} = 0.100$ kg; $m_3 = 200$ g $= 0.200$ kg; $x_2 = 0.20$ m; $x_3 = 0.80$ m.

SOLVE (a) Modeling the meterstick as a point object, the center of mass will be in the geometric center of the meterstick, or at the 0.500 m mark.

(b) With the two masses placed 0.200 m from each end of the meterstick,

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$X_{cm} = \frac{(0.120\text{ kg})(0.50\text{ m}) + (0.100\text{ kg})(0.20\text{ m}) + (0.200\text{ kg})(0.80\text{ m})}{0.120\text{ kg} + 0.100\text{ kg} + 0.200\text{ kg}} = 0.57\text{ m}$$

The center of mass is at the 57 cm mark.

REFLECT Each additional mass is 20 cm from opposite ends of the meterstick. It is reasonable that the center of mass would be closer to the heavier object.